Navigating Asymmetry

Insights from Aggregate and Choice Models on the Influence of Regular Prices and Discounts on Retailer Performances

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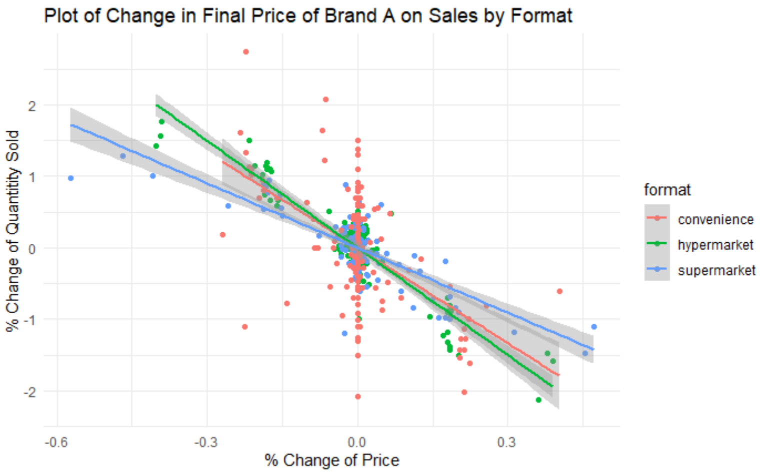
Executive Summary

- Most quantitative research papers model the effect of final retail price while recent research shows that neglecting the differences between regular prices and discounts may lead to biased estimates (i.e. overestimate/underestimate the effect)
- We conducted empirical testing for brand sales model specification and then employed the aggregate sales response model and individual choice model to estimate the price and discounts elasticities of brand and their potential asymmetricities between gains and losses
- It is necessary to consider appropriate model and phenomena regarding price to estimate the effect of price promotion on brand sales
  - Using appropriate model, the brand manager can design their prices and discounts offered that would benefit them based on store formats

Different Effects Between Final Price VS. Regular Price and Discounts

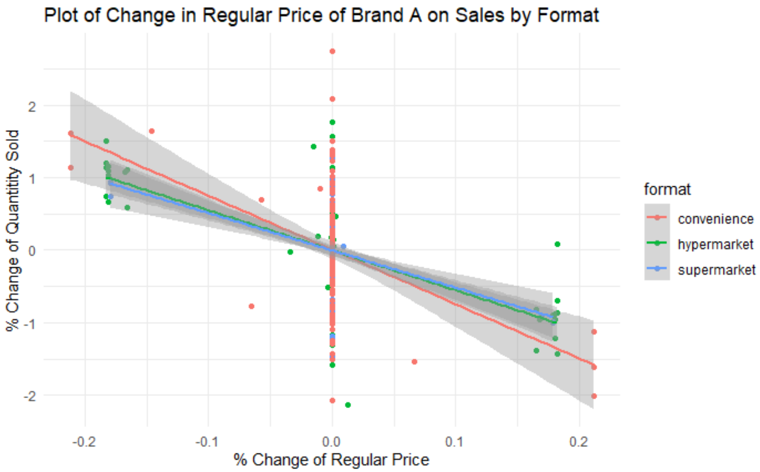
- Research in the area of consumer behavior suggests the potential promotion framing phenomena in which customers evaluate regular price and discounts differently
- Most quantitative research papers model the effect of price promotions as the effect of changes in the final retail price or the regular price
  - Maybe lack of discounts offered
  - Maybe information regarding discounts is not observable
- My previous findings indicate different discounts effectiveness (i.e. elasticities) across store formats

(Model-free) Evidence



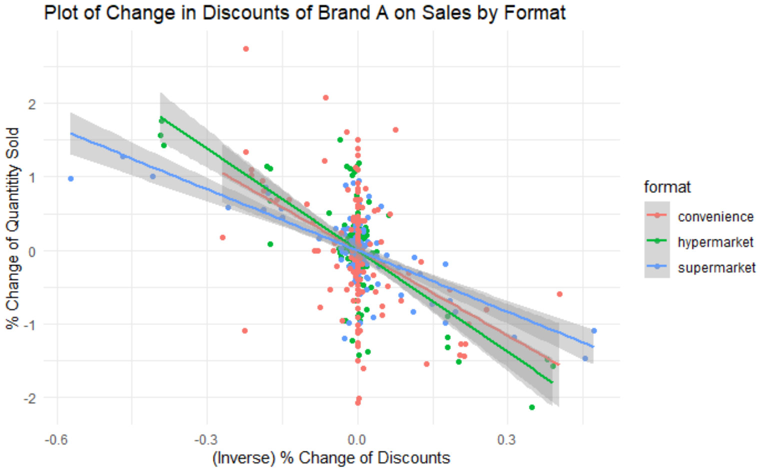
Brand A seems to have different effects of change in Price on different formats and differences between price increase and

(Model-free) Evidence



Brand A seems to have different effects of change in regular price on different formats and differences between its increase and decrease

(Model-free) Evidence



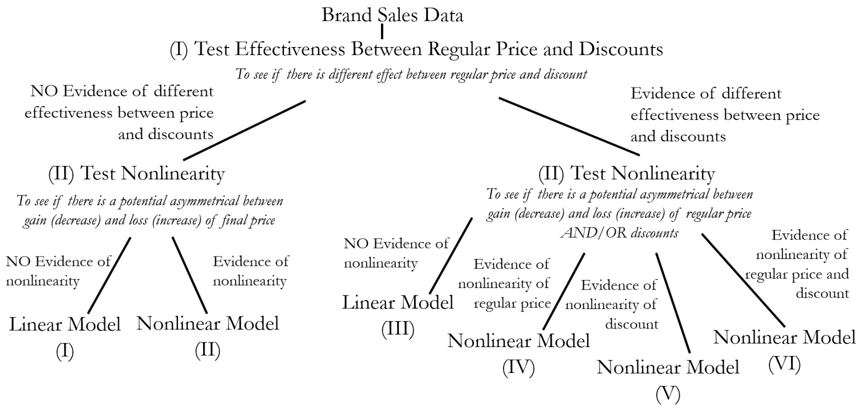
Brand A seems to have different effects of change in discounts on different formats and differences between its increase and decrease

Research Question

- How does the decomposition of promotional prices into regular prices and discounts affect sales' curves and the price elasticities?
- Are there asymmetric elasticities of gains and losses between regular prices and discounts?
- What are the potential price encoding mechanisms when customers made purchases across store formats?

To answer these questions, we will do empirical statistical tests on brand sales data

Empirical Testing for Brand Sales Model Specification



See more testing details and model specifications in Appendix

Testing Framework

- How does the decomposition of promotional prices into regular prices and discounts affect sales' curves and the price elasticities?
  - Test effectiveness between regular price and discounts to see if there is different effect between these two variables
- Are there asymmetric elasticities of gains and losses between regular prices and discounts?
  - Test for nonlinearity to see if there is a potential asymmetrical between gain and loss of final price, regular price and discounts
- What are the potential price encoding mechanisms when customers made purchases across store formats?
  - Employ the appropriate utility choice model to see how price-relevant information potentially affects brand choice

Data & Setting

- Point of sales data from loyalty cards from 10,0000 customers at one specific grocery hypermarket in a mid-sized city from Oct 2014 to Nov 2016
- We zoom in 3 major stores in the nearby location that our samples visit frequently
  - These stores include hypermarket format, supermarket format and convenience store format
- For this preliminary study, we zoom in one categories that more than 70 % of the total SKUs frequently purchased across three format
  - There are 11 brands offered across three store formats
- As we focus on brand and the retailer customers, we chose Brand B for an in-depth case analysis

Descriptive Statistics of Brand B (152 weeks observation for each format)

Statistic	N	Mean	St. Dev.	Min	Max
TotalSales(Unit)	456	107.78	77.83	2	301
Δ ln(Sales)	456	0.00	0.38	−1.44	1.70
Avg. Final Price	456	27.79	1.92	17.63	30.90
Δ ln(Final Price)	456	0.00	0.07	−0.29	0.47
Avg. Regular Price	456	28.11	1.64	23.45	30.90
Δ Regular Price	456	0.00	0.04	−0.19	0.19
Avg. Discount	456	0.32	0.87	0.00	8.44
Avg. Discount Depth	456	0.01	0.03	0.00	0.29
Δ ln(1-D)	456	0.00	0.05	−0.34	0.34
Avg. Competitive Price	456	21.96	2.08	16.30	28.31

Methodology: Test for Model Specification

- Test effectiveness between regular price and discounts:
    - Decompose the final price into regular price and discounts and test if their coefficients are statistically equivalent
- Test nonlinearity:
    - Add square term of relevant variable (i.e., final price, regular price and discounts) and test if these additional terms significantly improve the model fit
- Employ appropriate model:
    - Employ aggregate brand sales model to quantify the (a)symmetricity of elasticities between gain and loss
    - Employ individual choice model to quantify the (a)symmetricity of elasticities between gain and loss

See more test detail in Appendix

Results from Empirical Testing for Brand Sales Model Specification

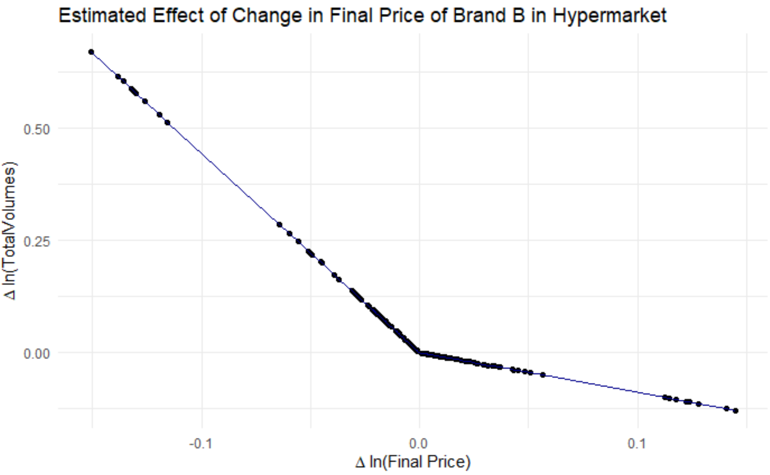
	Hypermarket	**Supermarket**	**Convenience Store**
Test Effectiveness between regular price and discounts	Insignificant difference of effect	Significant difference of effect	Insignificant difference of effect
Test Nonlinearity	Evidence of nonlinearity in final price	Evidence of nonlinearity in regular price	No evidence at all
Conclusion	Nonlinear model (II)	Nonlinear model (IV)	Linear model (I)

- Same brand requires different model specification implying different relevances of final price, regular price and discounts
- For utility choice model, the model with gain and loss asymmetry (in hypermarket and supermarket) in regular price will be employed

Aggregate Model (II) of Brand B Sales in Hypermarket

	$\Delta(ln(totalvolume))$
FinalPriceGain	-4.448^{***} (0.717)
FinalPriceLoss	-0.891 (1.572)
$\Delta ln(CompPrice)$	0.303 (0.223)
Constant	5.991 (4.542)
Observations	152
Adjusted R^2	0.751
Note:	$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Aggregate Model (II) of Brand B Sales in Hypermarket

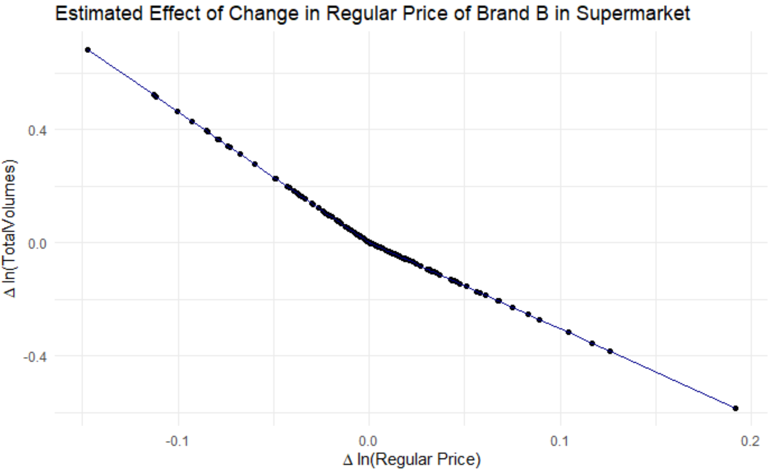


- Reduction in price is more effective

Aggregate Model (IV) of Brand B Sales in Supermarket

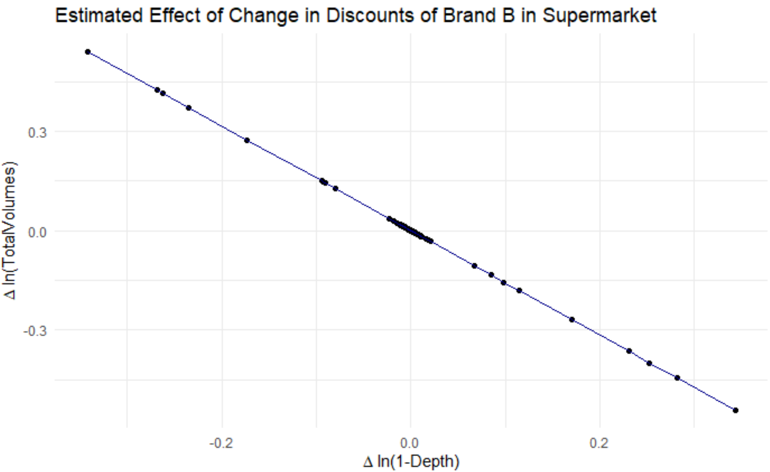
	$\Delta(ln(totalvolume))$
RegPriceGain	-4.626^{***} (0.788)
RegPriceLoss	-3.054^{***} (1.077)
$\Delta(1 - Depth)$	-1.578^{***} (0.269)
$\Delta ln(CompPrice)$	0.040 (0.179)
Constant	15.509^{***} (3.304)
Observations	152
Adjusted R^2	0.667
Note:	*p<0.1; **p<0.05; ***p<0.01

Aggregate Model (IV) of Brand B Sales in Supermarket



- Not much different between increase or decrease in regular price

Aggregate Model (IV) of Brand B Sales in Supermarket

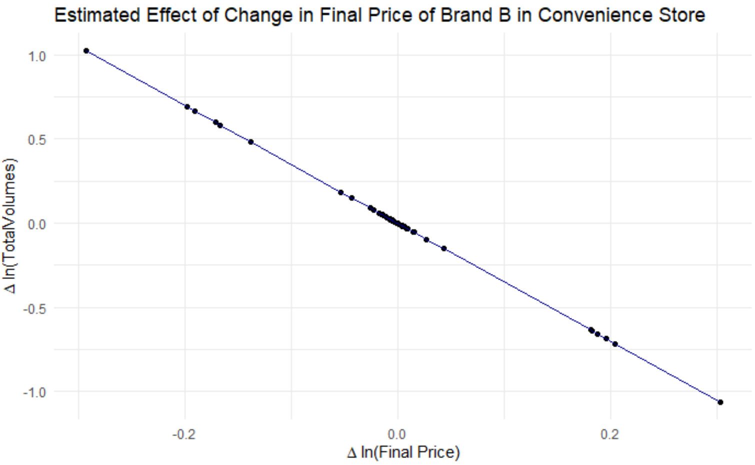


- Might be less effective compared to regular price

Aggregate Model (I) of Brand B Sales in Convenience Store

	$\Delta(ln(totalvolume))$
$\Delta ln(FinalPrice)$	-3.498^{***} (0.744)
$\Delta ln(CompPrice)$	0.399 (0.308)
Constant	15.184^{***} (3.617)
Observations	152
Adjusted R^2	0.564
Note:	*p<0.1; **p<0.05; ***p<0.01

Aggregate Model (I) of Brand B Sales in Convenience Store



Estimated Utility Choice Model of Brand B Across Formats

Regular Price Gain at Hypermarket	0.046*** (0.010)
Regular Price Loss at Hypermarket	0.032*** (0.009)
Regular Price Gain at Supermarket	0.046*** (0.011)
Regular Price Loss at Supermarket	−0.046*** (0.011)
Regular Price at Hypermarket[1]	−0.112*** (0.008)
Discounts Offered at Hypermarket[1]	0.115*** (0.012)
Regular Price at Supermarket[1]	−0.044*** (0.006)
Discounts Offered at Supermarket[1]	0.052*** (0.005)
Regular Price at Convenience[1]	−0.022*** (0.003)
Discounts Offered at Convenience[2]	0.012 (0.007)
NonfocalBrand [1]	−0.265*** (0.087)
Observations	130,846

[1]Random parameter with significant S.D.
[2]Random parameter with insignificant S.D.

Results

- It is necessary to consider appropriate model and phenomena regarding price to estimate the effect of price promotion on brand sales
- For our example brand (brand B), discounts seem to be relevant in supermarket format and customers are likely to be more responsive to an increase in final price (vs. decrease) in hypermarket format.
- Estimated choice model suggests different effects of price and promotion for Brand B across formats:
  - In general, hypermarket has highest price and discount elasticities
  - Customers are least sensitive to changes of prices and discounts in convenience store

Recommendation

- The brand manager can design their prices and discounts offered that would benefit them based on store formats
- For Brand B, the manager can focus on either offering more discounts or changing regular price in hypermarket format to attract customers
- For Brand B, the manager should focus on offering more discounts in supermarket format to attract customers
- For Brand B, the manager should focus on changing regular price in convenience store format to attract customers
- However, achieving higher volume sales does not necessarily mean achieving higher profit

Derivation of Regular Price and Discounts Specification

The natural logarithm model (Power Model) for brand sales i in category j in store format k is:

$$\begin{aligned}(1)\Delta ln(S_{ijk,t}) = {} & \beta_{1ijk} + \beta_{2ijk,t}\Delta ln(Price_{ijk,t}) + \beta_{3ijk}\Delta ln(CompPrice_{ijk,t}) \\ & + \gamma_i[S_{ijk,t-1} - \beta_{4ijk} ln(Price_{ijk,t-1})] \\ & + \beta_{5ijk} Holiday_t + Copula + \varepsilon_{ijk}\end{aligned}$$

We decompose the final price into regular price (or list price) and discount in the way that we want the argument of natural log to be positive.

$$FinalPrice_{ijk,t} = FinalPrice_{ijk,t} - FinalPrice_{ijk,t-1}$$

$$\Delta ln(FinalPrice_{ijk,t}) = ln(FinalPrice_{ijk,t}) - ln(FinalPrice_{ijk,t-1})$$

$$= ln(RegPrice_{ijk,t} - Disc_{ijk,t}) - ln(RegPrice_{ijk,t-1} - Disc_{ijk,t-1})$$

Derivation of Regular Price and Discounts Specification

$$
\begin{aligned}
&= \ln(\text{RegPrice}_{ijk,t} - \text{Discount}_{ijk,t}) - \ln(\text{RegPrice}_{ijk,t-1} - \text{Discount}_{ijk,t-1}) \\
&= \ln(\text{RegPrice}_{ijk,t}(1 - \frac{\text{Discount}_{ijk,t}}{\text{RegPrice}_{ijk,t}})) - \ln(\text{RegPrice}_{ijk,t-1}(1 - \frac{\text{Discount}_{ijk,t-1}}{\text{RegPrice}_{ijk,t-1}})) \\
&= \ln(\text{RegPrice}_{ijk,t}(1 - \text{Depth}_{ijk,t})) - \ln(\text{RegPrice}_{ijk,t-1}(1 - \text{Depth}_{ijk,t-1})) \\
&= \ln(\text{RegPrice}_{ijk,t}) + \ln(1 - \text{Depth}_{ijk,t}) - (\ln(\text{RegPrice}_{ijk,t-1}) + \ln(1 - \text{Depth}_{ijk,t-1})) \\
&= \ln(\text{RegPrice}_{ijk,t}) + \ln(1 - \text{Depth}_{ijk,t}) - \ln(\text{RegPrice}_{ijk,t-1}) - \ln(1 - \text{Depth}_{ijk,t-1})) \\
&= \ln(\text{RegPrice}_{ijk,t}) - \ln(\text{RegPrice}_{ijk,t-1}) + \ln(1 - \text{Depth}_{ijk,t}) - \ln(1 - \text{Depth}_{ijk,t-1})) \\
&= \Delta \ln(\text{RegPrice}_{ijk,t}) + \Delta \ln(1 - \text{Depth}_{ijk,t})
\end{aligned}
$$

- We then substitute $\Delta ln(FinalPrice_{ijk,t}) = \Delta \ln(\text{RegPrice}_{ijk,t}) + \Delta \ln(1 - \text{Depth}_{ijk,t})$ into (1)

Derivation of Regular Price and Discounts Specification

We get:

$$
\begin{aligned}
(2)\Delta ln(S_{ijk,t}) = {} & \beta_{1ijk} \\
& + \beta_{2ijk,t}\Delta ln(RegPrice_{ijk,t}) + \beta_{2'ijk,t}\Delta \ln(1 - \mathrm{Depth}_{ijk,t}) \\
& + \beta_{3ijk}\Delta ln(CompPrice_{ijk,t}) \\
& + \gamma_i[S_{ijk,t-1} - \beta_{4ijk} ln(RegPrice_{ijk,t-1}) - \beta_{4'ijk} ln(1 - Depth_{ijk,t-1})] \\
& + \beta_{5ijk} Holiday_t + Copula + \varepsilon_{ijk}
\end{aligned}
$$

Test Effectiveness Between Regular Price and Discounts

- From (aggregate) brand sales model

$$\begin{aligned}\Delta ln(S_{ijk,t}) = \beta_{1ijk} \\ &+ \beta_{2ijk,t}\Delta ln(RegPrice_{ijk,t}) + \beta_{2'ijk,t}\Delta \ln(1 - \text{Depth}_{ijk,t}) \\ &+ \beta_{3ijk}\Delta ln(CompPrice_{ijk,t}) \\ &+ \gamma_i[S_{ijk,t-1} - \beta_{4ijk}ln(RegPrice_{ijk,t-1}) - \beta_{4'ijk}ln(1 - Depth_{ijk,t-1} \\ &+ \beta_{5ijk}Holiday_t + Copula + \varepsilon_{ijk}\end{aligned}$$

- We test if $\beta_{2ijk,t} = \beta_{2'ijk,t}$ or $\beta_{2ijk,t} - \beta_{2'ijk,t} = 0$
  - Using the *linearHypothesis* function from the *car* package
  - More documentation available at Katherine S. Zee repository
- If we reject the null hypothesis, we continue using model (2) decomposing fina price into regular price and discounts, otherwise, use model (1) for nonlinearity test

Test Nonlinearity

- Following Pauwels et al. (2007), we will test "whether models with one or more transition functions are a useful way to fit the data (p. 90)" by estimating extended model (1) and (2) with cross products of $\Delta ln(FinalPrice_{ijk,t})$, $\Delta ln(RegPrice_{ijk,t})$, or $\Delta ln(Discount_{ijk,t})$ depending on evidence of different coefficients between regular price and discounts
- Thus, we have

$$\begin{aligned}(1.1)\Delta ln(S_{ijk,t}) = {} & \beta_{1ijk} + \beta_{2ijk,t}\Delta ln(FinalPrice_{ijk,t}) \\ & + \beta_{3ijk}\Delta ln(CompPrice_{ijk,t}) \\ & + \gamma_i[S_{ijk,t-1} - \beta_{4ijk} ln(Price_{ijk,t-1})] \\ & + \beta_{5ijk} Holiday_t + Copula + \varepsilon_{ijk} \\ & + \boldsymbol{\beta_{6ijk,t}\Delta ln(FinalPrice_{ijk,t})^2}\end{aligned}$$

Test Nonlinearity

$$
\begin{aligned}
(2.1)\Delta ln(S_{ijk,t}) = {} & \beta_{1ijk} \\
& + \beta_{2ijk,t}\Delta ln(RegPrice_{ijk,t}) + \beta_{2'ijk,t}\Delta\,\mathrm{ln}(1 - \mathrm{Depth}_{ijk,t}) \\
& + \beta_{3ijk}\Delta ln(CompPrice_{ijk,t}) \\
& + \gamma_i[S_{ijk,t-1} - \beta_{4ijk} ln(RegPrice_{ijk,t-1}) - \beta_{4'ijk} ln(Depth_{ijk,t-1})] \\
& + \beta_{5ijk} Holiday_t + Copula + \varepsilon_{ijk} \\
& + \boldsymbol{\beta_{6ijk,t}\Delta ln(RegPrice_{ijk,t})^2} \\
(2.2)\Delta ln(S_{ijk,t}) = {} & \beta_{0ijk} \\
& + \beta_{2ijk,t}\Delta ln(RegPrice_{ijk,t}) + \beta_{2'ijk,t}\Delta\,\mathrm{ln}(1 - \mathrm{Depth}_{ijk,t}) \\
& + \beta_{3ijk}\Delta ln(CompPrice_{ijk,t}) \\
& + \gamma_i[S_{ijk,t-1} - \beta_{4ijk} ln(RegPrice_{ijk,t-1}) - \beta_{4'ijk} ln(1 - Depth_{ijk,t-1} \\
& + \beta_{5ijk} Holiday_t + Copula + \varepsilon_{ijk} \\
& + \boldsymbol{\beta_{6ijk,t}\Delta ln(Discounts_{ijk,t})^2}
\end{aligned}
$$

Test Nonlinearity

$$
\begin{aligned}
(2.3)\Delta ln(S_{ijk,t}) = \beta_{1ijk} \\
&+ \beta_{2ijk,t}\Delta ln(RegPrice_{ijk,t}) + \beta_{2'ijk,t}\Delta \ln(1-\text{Depth}_{ijk,t}) \\
&+ \beta_{3ijk}\Delta ln(CompPrice_{ijk,t}) \\
&+ \gamma_i[S_{ijk,t-1} - \beta_{4ijk} ln(RegPrice_{ijk,t-1}) - \beta_{4'ijk} ln(1 - Depth_{ijk,t-1}) \\
&+ \beta_{5ijk} Holiday_t + Copula + \varepsilon_{ijk} \\
&+ \boldsymbol{\beta_{6ijk,t}\Delta ln(RegPrice_{ijk,t})^2 + \beta_{7ijk,t}\Delta ln(Discounts_{ijk,t})^2}
\end{aligned}
$$

- We compare model (1.1) with (1) and compare model (2.1), (2.2) and (2.3) with (2) using Likelihood ratio to test the relevance of the additional variable(s)
- Test if $\beta_{6ijk,t} = 0$ in (1.1), (2.1), (2.2)
- Test if $\beta_{6ijk,t} = \beta_{7ijk,t} = 0$ in (2.3)

Test Nonlinearity

- Linear Model (I): If $\beta_{6ijk,t}$ in (1.1) is not relevant
- Nonlinear Model (II): If $\beta_{6ijk,t}$ in (1.1) is relevant
- Linear Model (III): If $\beta_{6ijk,t}$ and $\beta_{7ijk,t}$ in (2.1), (2.2) and (2.3) are not relevant
- Nonlinear Model (IV): If $\beta_{6ijk,t}$ in (2.1) is relevant but $\beta_{6ijk,t}$ and $\beta_{7ijk,t}$ in (2.2) and (2.3) are not relevant
- Nonlinear Model (V): If $\beta_{6ijk,t}$ in (2.2) is relevant but $\beta_{6ijk,t}$ and $\beta_{7ijk,t}$ in (2.1) and (2.3) are not relevant
- Nonlinear Model (VI): If $\beta_{6ijk,t}$ and $\beta_{7ijk,t}$ in (2.1) and (2.3) are relevant

Linear Model (I)

- Aggregate Model:

$$\begin{aligned}\Delta ln(S_{ijk,t}) = &\ \beta_{1ijk} + \beta_{2ijk,t}\Delta ln(FinalPrice_{ijk,t}) \\ &+ \beta_{3ijk}\Delta ln(CompPrice_{ijk,t}) \\ &+ \gamma_i[S_{ijk,t-1} - \beta_{4ijk} ln(FinalPrice_{ijk,t-1})] \\ &+ \beta_{5ijk} Holiday_t + Copula + \varepsilon_{ijk}\end{aligned}$$

Nonlinear Model (II)

- Aggregate Model which can be estimated using MLE:

$$
\begin{aligned}
\Delta ln(S_{ijk,t}) = {} & \beta_{1ijk} \\
& + \left[\alpha_0 + \frac{\alpha_{L,P}}{1+\exp(-\gamma \Delta ln(FinalPrice_{ijk,t})}\right](\Delta \ln(\mathrm{FinalPrice}_{ijk,t})) \\
& + \beta_{3ijk}\Delta ln(CompPrice_{ijk,t}) \\
& + \gamma_i[S_{ijk,t-1} - \beta_{4ijk} ln(FinalPrice_{ijk,t-1})] \\
& + \beta_{5ijk} Holiday_t + Copula + \varepsilon_{ijk}
\end{aligned}
$$

where $\alpha_{L,P}$ indicating the change of (final price) elasticity from gain (α_0) and γ is the smoothness of the transition curve reflecting how fast the coefficient of gain changes to loss

Nonlinear Model (II)

- Simplified Form of Aggregate Model ($\gamma \to \infty$) using OLS:

$$\begin{aligned}\Delta ln(S_{ijk,t}) = \beta_{1ijk} \\ &+ \beta_{2Gijk} FinalPriceGain + \beta_{2Lijk} FinalPriceLoss \\ &+ \beta_{3ijk} \Delta ln(CompPrice_{ijk,t}) \\ &+ \gamma_i [S_{ijk,t-1} - \beta_{4ijk} ln(FinalPrice_{ijk,t-1})] \\ &+ \beta_{5ijk} Holiday_t + Copula + \varepsilon_{ijk}\end{aligned}$$

  where
  $FinalPriceGain = \Delta \ln(\mathrm{FinalPrice}_{ijk,t}) if \Delta \ln(\mathrm{FinalPrice}_{ijk,t}) < 0$, else 0
  $FinalPriceLoss = \Delta \ln(\mathrm{FinalPrice}_{ijk,t}) if \Delta \ln(\mathrm{FinalPrice}_{ijk,t}) > 0$, else 0

- In this case, β_{2G} is equivalent to α_0 and β_{2L} is equivalent to $\alpha_0 + \alpha_{L,P}$
- Note: For this case, we are interested in the final coefficient, not the difference per se

Linear Model (III)

- Aggregate Model:

$$
\begin{aligned}
\Delta ln(S_{ijk,t}) = {} & \beta_{1ijk} \\
& + \beta_{2ijk,t} \Delta ln(RegPrice_{ijk,t}) + \beta_{3ijk,t} \Delta \ln(1 - \mathrm{Depth}_{ijk,t}) \\
& + \beta_{4ijk} \Delta ln(CompPrice_{ijk,t}) \\
& + \gamma_i [S_{ijk,t-1} - \beta_{5ijk} ln(RegPrice_{ijk,t-1}) - \beta_{6' ijk} ln(1 - Depth_{ijk,t-1} \\
& + \beta_{7ijk} Holiday_t + Copula + \varepsilon_{ijk}
\end{aligned}
$$

Nonlinear Model (IV)

- Aggregate Model which can be estimated using MLE:

$$
\begin{aligned}
\Delta ln(S_{ijk,t}) = \beta_{1ijk} \\
&+ \left[\alpha_{0,P} + \frac{\alpha_{L,P}}{1+\exp(-\gamma \Delta ln(RegPrice_{ijk,t}))}\right] (\Delta \ln(\mathrm{RegPrice}_{ijk,t})) \\
&+ \beta_{3ijk} \Delta ln(1 - Depth_{ijk,t}) \\
&+ \beta_{4ijk} \Delta ln(CompPrice_{ijk,t}) \\
&+ \gamma_i [S_{ijk,t-1} - \beta_{5ijk} ln(FinalPrice_{ijk,t-1})] \\
&+ \beta_{6ijk} Holiday_t + Copula + \varepsilon_{ijk}
\end{aligned}
$$

where $\alpha_{L,P}$ indicating the change of (regular price) elasticity from gain ($\alpha_{0,P}$) and γ is the smoothness of the transition curve reflecting how fast the coefficient of gain changes to loss

Nonlinear Model (IV)

- Simplified Form of Aggregate Model ($\gamma \to \infty$) using OLS:

$$\begin{aligned}\Delta ln(S_{ijk,t}) = {} & \beta_{1ijk} \\ & + \beta_{2Gijk} RegPriceGain + \beta_{2Lijk} RegPriceLoss \\ & + \beta_{3ijk} \Delta ln(1 - Depth_{ijk,t}) \\ & + \beta_{4ijk} \Delta ln(CompPrice_{ijk,t}) \\ & + \gamma_i [S_{ijk,t-1} - \beta_{4ijk} ln(RegPrice_{ijk,t-1}) - \beta_{4' ijk} ln(1 - Depth_{ijk,t-1} \\ & + \beta_{6ijk} Holiday_t + Copula + \varepsilon_{ijk}\end{aligned}$$

where
$RegPriceGain = \Delta \ln(\text{RegPrice}_{ijk,t}) if \Delta \ln(\text{RegPrice}_{ijk,t}) < 0$, else 0
$RegPriceLoss = \Delta \ln(\text{RegPrice}_{ijk,t}) if \Delta \ln(\text{RegPrice}_{ijk,t}) > 0$, else 0

- In this case, β_{2G} is equivalent to α_0 and β_{2L} is equivalent to $\alpha_{0,P} + \alpha_{L,P}$

Nonlinear Model (V)

- Aggregate Model which can be estimated using MLE:

$$
\begin{aligned}
\Delta ln(S_{ijk,t}) = \beta_{1ijk} \\
& + \beta_{2ijk}\Delta ln(RegPrice_{ijk,t-1}) \\
& + \left[\alpha_{0,D} + \frac{\alpha_{L,D}}{1+\exp(-\gamma\Delta ln(1-Depth_{ijk,t}))}\right](\Delta \ln(1-Depth_{ijk,} \\
& + \beta_{4ijk}\Delta ln(CompPrice_{ijk,t}) \\
& + \gamma_i[S_{ijk,t-1} - \beta_{4ijk} ln(RegPrice_{ijk,t-1}) - \beta_{4' ijk} ln(1-Depth_{ijk,t-1} \\
& + \beta_{6ijk} Holiday_t + Copula + \varepsilon_{ijk}
\end{aligned}
$$

where $\alpha_{L,D}$ indicating the change of (discounts) elasticity from gain ($\alpha_{0,D}$) and γ is the smoothness of the transition curve reflecting how fast the coefficient of gain changes to loss

Nonlinear Model (V)

- Simplified Form of Aggregate Model ($\gamma \to \infty$) using OLS:

$$\begin{aligned}
\Delta ln(S_{ijk,t}) = \beta_{1ijk} \\
&+ \beta_{2ijk} \Delta ln(RegPrice_{ijk,t}) \\
&+ \beta_{3Gijk} NonDepthGain + \beta_{3Lijk} NonDepthLoss \\
&+ \beta_{4ijk} \Delta ln(CompPrice_{ijk,t}) \\
&+ \gamma_i [S_{ijk,t-1} - \beta_{4ijk} ln(RegPrice_{ijk,t-1}) - \beta_{4'ijk} ln(1 - Depth_{ijk,t-1} \\
&+ \beta_{6ijk} Holiday_t + Copula + \varepsilon_{ijk}
\end{aligned}$$

where

$NonDepthGain = \Delta \ln(\text{1-Depth}_{ijk,t}) if \Delta \ln(\text{1-Depth}_{ijk,t}) < 0$, else 0

$NonDepthLoss = \Delta \ln(\text{1-Depth}_{ijk,t}) if \Delta \ln(\text{1-Depth}_{ijk,t}) > 0$, else 0

- In this case, β_{3G} is equivalent to $\alpha_{0,D}$ and β_{3L} is equivalent to $\alpha_{0,D} + \alpha_{L,D}$

Nonlinear Model (VI)

- Aggregate Model which can be estimated using MLE:

$$\begin{aligned}\Delta ln(S_{ijk,t}) &= \beta_{1ijk} \\ &+ \left[\alpha_{0,P} + \frac{\alpha_{L,P}}{1+\exp(-\gamma\Delta ln(RegPrice_{ijk,t})}\right](\Delta\,\mathrm{ln}(\mathrm{RegPrice}_{ijk,t})) \\ &+ \left[\alpha_{0,D} + \frac{\alpha_{L,D}}{1+\exp(-\gamma\Delta ln(1-Depth_{ijk,t}))}\right](\Delta\,\mathrm{ln}(1-Depth_{ijk,} \\ &+ \beta_{4ijk}\Delta ln(CompPrice_{ijk,t}) \\ &+ \gamma_i[S_{ijk,t-1} - \beta_{4ijk}ln(RegPrice_{ijk,t-1}) - \beta_{4'ijk}ln(1-Depth_{ijk,t-1} \\ &+ \beta_{6ijk}Holiday_t + Copula + \varepsilon_{ijk}\end{aligned}$$

Nonlinear Model (VI)

- Simplified Form of Aggregate Model ($\gamma \to \infty$) using OLS:

$$
\begin{aligned}
\Delta ln(S_{ijk,t}) = \ & \beta_{1ijk} \\
& + \beta_{2Gijk} RegPriceGain + \beta_{2Lijk} RegPriceLoss \\
& + \beta_{3Gijk} NonDepthGain + \beta_{3Lijk} NonDepthLoss \\
& + \beta_{4ijk} \Delta ln(CompPrice_{ijk,t}) \\
& + \gamma_i [S_{ijk,t-1} - \beta_{4ijk} ln(RegPrice_{ijk,t-1}) - \beta_{4'ijk} ln(1 - Depth_{ijk,t-1} \\
& + \beta_{6ijk} Holiday_t + Copula + \varepsilon_{ijk}
\end{aligned}
$$

(Utility) Choice Model with Gain/Loss Symmetry

Following Elshiewy and Perschel (2021); the utility (U) of household h, to choose brand i in category j, in choice situation t across different formats (hypermarket, supermarket, convenience store) (with error e_{hijt}):

$$\begin{aligned} U_{hijt} &= V_{hijt} + e_{hijt} \\ &= \beta_{\text{BLOY}} \text{BLOY}_{ijt} + \beta_{\text{Competitive Brand(s)}_{hjt}} \\ &+ \beta_{\text{hyperprice}} \text{HyperPrice}_{hijt} + \beta_{\text{hyperdis}} \text{HyperDis}_{hijt} \\ &+ \beta_{\text{superprice}} \text{SuperPrice}_{hijt} + \beta_{\text{superdis}} \text{SuperDis}_{hijt} \\ &+ \beta_{\text{Conveprice}} \text{ConvePrice}_{hijt} + \beta_{\text{convedis}} \text{ConveDis}_{hijt} + e_{hijt} \end{aligned}$$

where $BLOY_{ijt}$ is the brand-specific loyalty measure (Guadagni & Little, 2008), $\beta_{\text{Competitive Brand(s)}}$ is competitive brand intercept (as compared to focal brand) and parameters β_h is an individual parameter to account for customer response heterogeneity

(Utility) Choice Model with Gain/Loss Asymmetry

Following Elshiewy and Perschel (2021); the utility (U) of household h, to choose brand i in category j, in choice situation t across different formats (hypermarket, supermarket, convenience store) (with error e_{hijt}):

$$
\begin{aligned}
U_{hijt} = {} & V_{hijt} + e_{hijt} \\
& = \beta_{\text{hyperpricegain}} \text{HyperPriceGain}_{ijt} + \beta_{\text{hyperpriceloss}} \text{HyperPriceLoss}_{ijt} \\
& + \beta_{\text{hyperdisgain}} \text{HyperDisGain}_{ijt} + \beta_{\text{hyperdisloss}} \text{HyperDisLoss}_{ijt} \\
& + \beta_{\text{superpricegain}} \text{SuperPriceGain}_{ijt} + \beta_{\text{superpriceloss}} \text{SuperPriceLoss}_{ijt} \\
& + \beta_{\text{superdisgain}} \text{SuperDisGain}_{ijt} + \beta_{\text{superdisloss}} \text{SuperDisLoss}_{ijt} \\
& + \beta_{\text{Convepricegain}} \text{ConvePriceGain}_{ijt} + \beta_{\text{Convepriceloss}} \text{ConvePriceLoss}_{ijt} \\
& + \beta_{\text{Convedisgain}} \text{ConveDisGain}_{ijt} + \beta_{\text{Convedisloss}} \text{ConveDisLoss}_{ijt} \\
& + \beta_{\text{BLOY}} \text{BLOY}_{ijt} + \beta_{\text{Competitive Brand}_{hjt}} \\
& + \beta_{\text{hyperprice}} \text{HyperPrice}_{hijt} + \beta_{\text{hyperdis}} \text{HyperDis}_{hjt} \\
& + \beta_{\text{superprice}} \text{SuperPrice}_{hijt} + \beta_{\text{superdis}} \text{SuperDis}_{hijt} \\
& + \beta_{\text{Conveprice}} \text{ConvePrice}_{hijt} + \beta_{\text{convedis}} \text{ConveDis}_{hijt} + e_{hijt}
\end{aligned}
$$

(Utility) Choice Model with Gain/Loss Asymmetry

- Gain and loss of regular price and discounts variables across formats (*PriceGain*, *PriceLoss*, *DisGain*, *DisLoss*) are operationalized at the household level
- Hence, gain and loss are deviations from previous prices and discounts the household h encountered at a specific format which become their internal reference price (IRP) and internal reference discount (IRD)
  - We defined IRP according to Elshiewy and Perschel (2021):

$$IRP_{ij} = \lambda \cdot IRP_{hij,t-1} + (1-\lambda) \cdot PRICE_{hij,t-1}$$

where $PRICE_{hij,t-1}$ is the regular price observed for the brand in the last choice situation and λ is the smoothness of past prices

- E.g., $HyperPricegain = (IRP_{hij,t-1} - Price_{hij,t-1})$, if $(IRP_{hij,t-1} > Price_{hij,t-1})$ and t at Hypermarket, else 0

(Utility) Choice Model

- Following Elshiewy and Perschel (2021), we assume the error e_{hijt} to follow an i.i.d. Type I Extreme Value distribution. As a result, the choice probability P of consumer h, to choose brand i of category j, in choice situation t becomes the Multinomial Logit (MNL) formula (Train, 2009, p. 36):

$$P_{njt} = \frac{\exp(V_{hijt})}{\sum_{j=1}^{J} \exp(V_{hijt})}$$

- The proposed choice model already decomposed final price into regular price and discount, the model can be specified only final price instead and we may get:

$$\begin{aligned} U_{hijt} = {} & \beta_{\text{BLOY}} \text{BLOY}_{ijt} + \beta_{\text{Competitive Brand(s)}_{hjt}} \\ & + \beta_{\text{hyperfinalprice}} \text{HyperFinalPrice}_{hijt} \\ & + \beta_{\text{superfinalprice}} \text{SuperFinalPrice}_{hijt} \\ & + \beta_{\text{Convefinalprice}} \text{ConveFinalPrice}_{hijt} + e_{hijt} \end{aligned}$$

Extended Aggregate Nonlinear Model

One can consider (different types of) prices and discounts (gain and loss) threshold in aggregate model as proposed by Pauwels et al. (2007):

$$\begin{aligned}
\Delta \ln(S_{ijk,t}) = \beta_{1ijk} & \\
& + [\alpha_0 + \frac{\alpha_{G,P}}{1+\exp(\gamma(\Delta \ln(P_{ijk,t})) - \phi_{G,P})} \\
& + \frac{\alpha_{L,P}}{1+\exp(-\gamma(\Delta \ln(P_{ijk,t})) - \phi_{L,P})}]\Delta \ln(P_{ijk,t}) \\
& + [\alpha'_0 + \frac{\alpha'_{G,D}}{1+\exp(\gamma(\Delta \ln(1-D_{ijk,t})) - \phi'_{G,D})} \\
& + \frac{\alpha'_{L,D}}{1+\exp(-\gamma(\Delta \ln(1-D_{ijk,t})) - \phi'_{L,D})}]\Delta \ln(1-D_{ijk,t}) \\
& + \beta_{4ijk}\Delta \ln(\text{CompPrice}_{ijk,t}) \\
& + \gamma_i \left[S_{ijk,t-1} - \beta_{4ijk}\ln(P_{ijk,t-1}) - \beta_{4'ijk}\ln(1-D_{ijk,t-1})\right] \\
& + \beta_{6ijk}\text{Holiday}_t + \text{Copula} + \varepsilon_{ijk},
\end{aligned}$$

This extended model, which can be estimated using MLE, captures different thresholds (ϕ_L, ϕ_G) indicating the points at which elasticity changes and explicitly separate the elasticity into three regimes($\alpha_0, \alpha_0 + \alpha_G, \alpha_0 + \alpha_L$).